Electronic and Signal Processing Exam

Prof. Dr. E. Chicca

Exam - April 4, 2023

Information

Welcome to the Electronics and Signal Processing exam. Please read carefully the information below.

How to write your solution
Please use a **pen** and not a pencil. Make sure your **hand writing** is understandable by others. **Drawings** do not need to be beautiful/perfect but it is important that they are easy to understand and there are no ambiguities (e.g. a gate which could be an OR or an AND, but it is not clear which one it is, label it to avoid confusion).
Each **solution** has to be justified and **the steps to get there have to be explicitly written down**, only providing the final outcome will lead to zero points. On **every page** please indicate which problem you are working on. If you separate the pages please indicate your name and student ID on **every page**.
For your convenience, you can find a page with basic equations related to the course material at the end of this document.

Problem 1 (12 points)

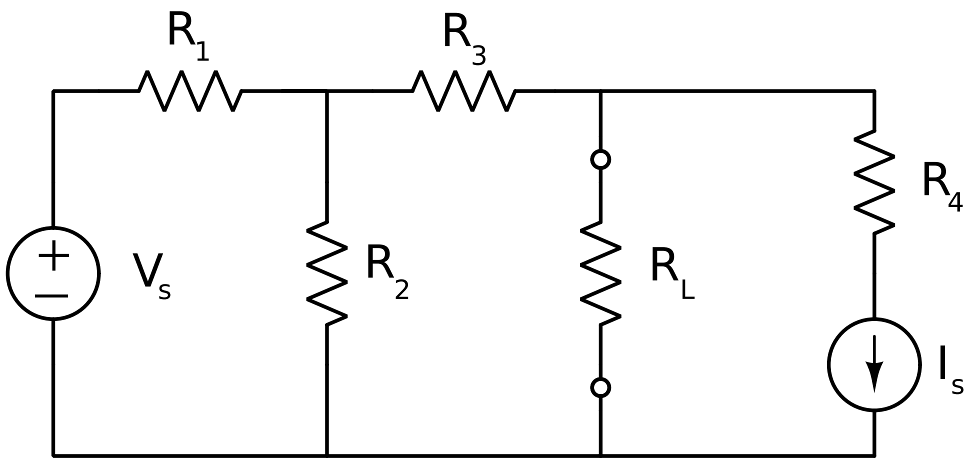


Figure 1: Resistor network.

Consider the circuit in Figure 1 and the related **parameters**: $R_1 = 4\ k\Omega$, $R_2 = 4\ k\Omega$, $R_3 = 2\ k\Omega$, $R_4 = 2\ k\Omega$, $V_s = 12\ V$, $I_s = 1\ mA$.

(a) (4 points) Using the parameters provided above, calculate the equivalent resistance R_{eq} seen by R_L (consider R_L an open circuit and calculate the resistance between the two open nodes).

(b) (5 points) Using the parameters provided above, calculate the Thévenin equivalent voltage V_{th} seen by R_L.

(c) (1 points) Draw the Thévenin equivalent circuit including R_L as load.

(d) (2 points) Given that $R_L = 6k\Omega$, calculate the current flowing through and the voltage across the load resistance R_L. If you do not have values for V_{th} and R_{eq} calculated from the previous questions use the following **incorrect** values: $V_{th} = 1\ V$ and $R_{eq} = 1\ k\Omega$.

Problem 1 - Solution

Point a)

We redraw the circuit as seen in Fig. 2. If we now replace the generators with their internal resistors, (that means the voltage source acts as a short circuit and the current source acts as an open circuit) we see that the equivalent resistance seen by R_L is:

$$R_{eq} = R_3 + R_2//R_1 = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 2\text{k}\Omega + \frac{4 \cdot 4}{4+4}\ \text{k}\Omega$$

So $R_{eq} = \underline{\underline{4\text{k}\Omega}}$.

Alternatively, one could notice that $R_1 = R_2 = 2R$ and $R_3 = R_4 = R = 2\ \Omega$:

$$R_{eq} = R_3 + R_2//R_1 = R + 2R//2R = R + R = 4\ \text{k}\Omega$$

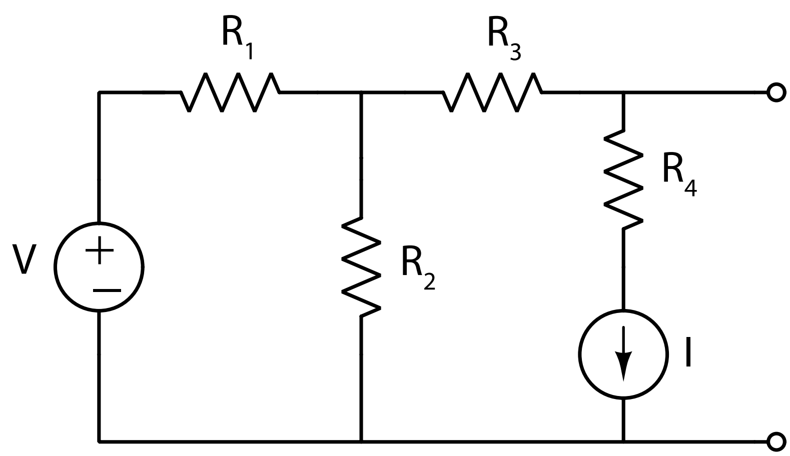


Figure 2: Thevenin Equivalent R1: Step 1.

Point b)

Solution I

The equivalent generator can be calculated using the superposition principle. Replacing the current source with an open circuit as in figure 3.V_{OCV}, the current through R_3 and R_4 must be zero, as R_4 leads to a dead end. This means that the contribution of the voltage source can be derived thanks to the potential divider equation:

$$V_{\text{OCV}} = \frac{R_2}{R_1 + R_2} \cdot V_S = \frac{4}{4+4} \cdot 12\text{V} = 6\text{V}$$

Alternatively, using notation introduced in the solution of a)

$$V_{\text{OCV}} = \frac{R_2}{R_1 + R_2} \cdot V_S = \frac{2R}{4R} \cdot V_S = \frac{1}{2} \cdot 12\text{V} = 6\text{V}$$

To find the contribution from the current source, we replace the voltage source with a short-circuit see figure 3.V_{OCI}. The open circuit voltage is given by

$$V_{\text{OCI}} = -I(R_3 + R_1//R_2) = -1\text{mA} \cdot (2 + 4//4)\text{k}\Omega = -4\text{V}.$$

By the superposition principle, the equivalent voltage seen by the resistor is the sum of the voltages calculated above, so

$$V_{\mathrm{OC}} = V_{\mathrm{OCI}} + V_{\mathrm{OCV}} = -4\mathrm{V} + 6\mathrm{V} = 2\mathrm{V}$$

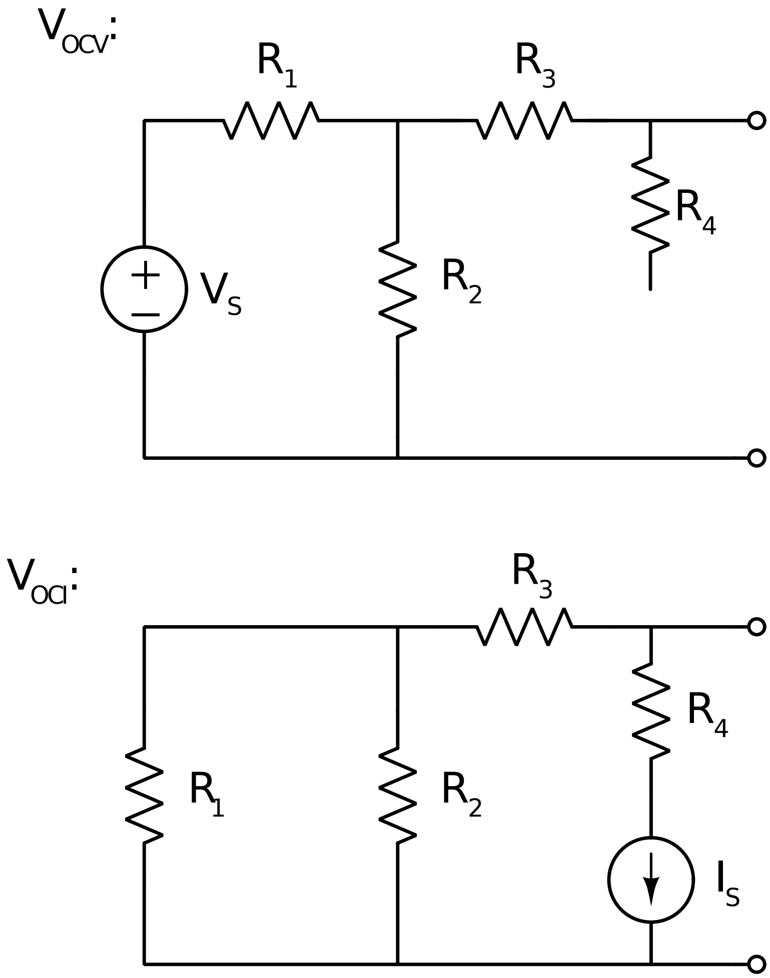


Figure 3: Superposition principle.

Solution II

Alternatively nodal analysis can be used to derive V_{OC}. Let us first apply KCL at the node where R_1, R_2 and R_3 are connected, from now on referred as V_x:

$$\frac{V_x - V_S}{2R} + \frac{V_x}{2R} + \frac{V_x - V_{OC}}{R} = 0$$

$$V_x \left(\frac{1}{2} + \frac{1}{2} + 1 \right) = \frac{V_S}{2} + V_{OC}$$

$$V_x = \frac{V_S}{4} + \frac{V_{OC}}{2}$$

Now we apply KCL to V_{OC}:

$$\frac{V_{OC} - V_x}{R} + I_S = 0$$

$$V_{OC} - V_x + RI_S = 0$$

$$V_x = V_{OC} + RI_S$$

We can put the two expressions for V_x together:

$$V_{OC} + RI_S = \frac{V_S}{4} + \frac{V_{OC}}{2}$$

$$\frac{V_{OC}}{2} = \frac{V_S}{4} - RI_S$$

$$V_{OC} = \frac{V_S}{2} - 2RI_S = 6\ V - 2 \cdot 2\ V = 2\ V$$

Solution III

Alternatively we can derive the short circuit current I_{SC} and use it to calculate the open circuit voltage $V_{OC} = I_{SC}R_{eq}$. We use again the superposition principle. When there is only the voltage source, we need to compute the current flowing through R_3. The voltage V_x across R_3 can be written using the potential divider equation:

$$V_x = \frac{R_3//R_2}{R_1 + R_3//R_2} V_S = \frac{\frac{2}{3}R}{2R + \frac{2}{3}R} V_S = \frac{V_S}{4}$$

$$I_{SCV} = \frac{V_x}{R_3} = \frac{V_s}{4R} = \frac{12\ V}{4 \cdot 2\ k\Omega} = 1.5\ mA$$

When there is only the current source $I_{SCI} = -1\ mA$.
Adding these two results together,

$$I_{SC} = I_{SCI} + I_{SCV} = 1.5 - 1\text{mA} = 0.5\text{mA}$$

Last, we find the equivalent voltage

$$V_{SC} = I_{SC}R_{eq} = 0.5 \cdot 4\ k\Omega = 2\ V$$

Point c)

We can use the answers from points a) and b) to redraw the circuit as shown in Figure 4. The sources and resistors (except for R_L) have been replaced by a single voltage source in series with a single resistor such that the voltage between the nodes around R_L is equivalent to the original situation.

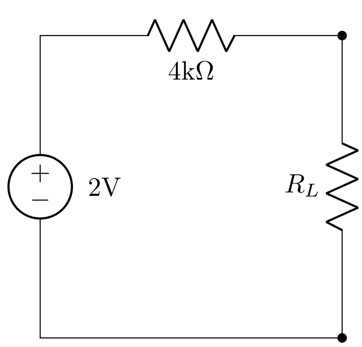


Figure 4: Thevenin equivalent circuit

Point d)

The total resistance is made up of the equivalent resistance $R_{eq} = 4\text{k}\Omega$ and the load resistance $R_L = 6\text{k}\Omega$ connected in series, yielding

$$R_{tot} = 4\text{k}\Omega + 6\text{k}\Omega = 10\text{k}\Omega.$$

This sets the current through the circuit at

$$I = I_L = 2\text{V}/10\text{k}\Omega = 0.2\text{mA},$$

making the voltage over R_L be

$$V_L = R_L I = 6\text{k}\Omega \cdot 0.2\text{mA} = 1.2\text{V}.$$

Remarks

This question is comparable to the Top Problem from Week 2. The number and type of sources and resistors is identical (1 voltage source and 1 current source, 4 resistors).

Problem 2 (17 points)

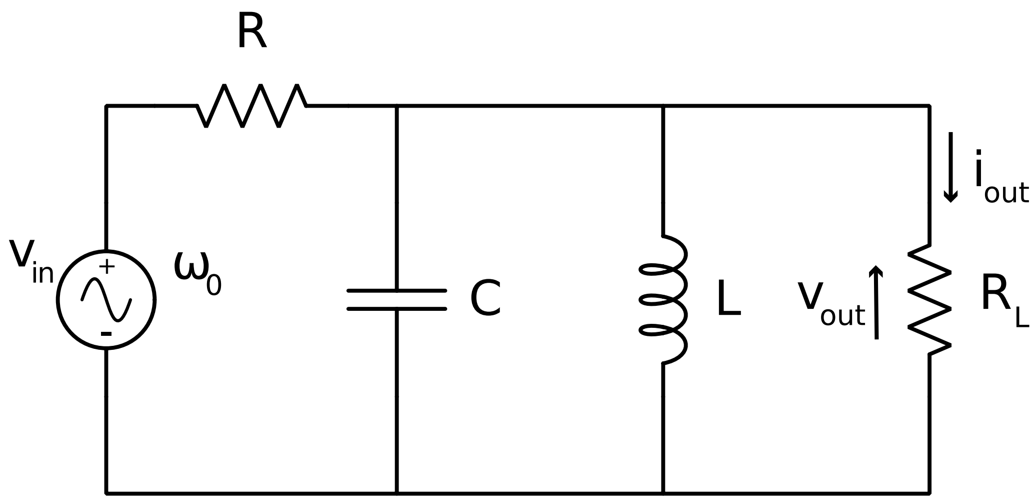


Figure 5: RLC circuit.

Consider the circuit in Figure 5. Assume **sinusoidal** regime, **ideal** components and the following **parameters**: $C = 0.7\ F$, $L = 0.5\ H$, $R = 1\ \Omega$, $R_L = 5\ \Omega$, $\omega_0 = 2$ rad s^{-1}.

(a) (3 points) **Without** any calculation, but **only** reasoning about the behavior of each element in the circuit, describe the behavior of

$$H(\omega) = \frac{i_{\text{out}}}{v_{\text{in}}}$$

for low ($\omega \to 0$) and high ($\omega \to \infty$) frequencies.

(b) (8 points) Using the parameters provided above, calculate the transfer function $H(\omega_0)$.

You may assume as true that $H(\omega = 1 \text{ rad } s^{-1}) = 0.11 + 0.75j\ AV^{-1}$ and $H(\omega = 3 \text{ rad } s^{-1}) = 0.07 - 0.08j\ AV^{-1}$.

(c) (3 points) Assuming $v_{\text{in}}(t) = -\sqrt{2}\sin(3t)\ V$, determine $i_{\text{out}}(t)$, giving your answer in the form $|I|\cos(\omega t + \phi)$.

(d) (3 points) Assume instead that $v_{\text{in}}(t) = 2\cos(3t) + \cos(t)\ V$. Using the superposition principle, find z.

Problem 2 - Solution

Point a)

For low frequencies ($\omega \to 0$ rad/s) the capacitor impedance $Z_C \to \infty$, while the inductor impedance $Z_L \to 0$. Thus the capacitor may be replaced by an open circuit, and the inductor by a short circuit. Thus all current flows through the inductor and $i_{out} = 0$ and so $\underline{\underline{H(j\omega) \to 0}}$.

For high frequencies ($\omega \to \infty$ rad/s) the capacitor impedance $Z_C \to 0$, while the inductor impedance $Z_L \to \infty$. Thus the capacitor may be replaced by a short circuit, and the inductor by an open circuit. Thus all current flows through the capacitor and $i_{out} = 0$ and so $\underline{\underline{H(j\omega) \to 0}}$.

Point b)

Solution I

We first calculate the impedance of the 3 parallel components.

$$\begin{aligned} Z_\parallel &= \left[Z_C^{-1} + Z_L^{-1} + Z_{R_L}^{-1}\right]^{-1} = \left[j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}\right]^{-1} \\ &= \left[1.4j - 1j + 0.2\right]^{-1}\Omega = \left[0.2 + 0.4j\right]^{-1}\Omega \end{aligned}$$

$$\boxed{Z_\parallel = (1 - 2j)\ \Omega}$$

Our effective circuit is then the resistor R in series with $Z_\parallel$. We apply the voltage divider equation to find the voltage across $Z_\parallel$

$$v_{\text{out}} = v_\parallel = \frac{Z_\parallel}{Z_\parallel + R} v_{\text{in}} = \frac{1 - 2j}{1 - 2j + 1} v_{\text{in}} = \frac{1 - 2j}{2 - 2j} v_{\text{in}}$$

$$\boxed{v_{\text{out}} = (0.75 - 0.25j) v_{\text{in}}}$$

We can then derive $H(\omega_0)$:

$$\begin{aligned} H(\omega_0) &= \frac{i_{\text{out}}}{v_{\text{in}}} = \frac{1}{R_L}\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{R_L}(0.75 - 0.25j) = \frac{1}{5}(0.75 - 0.25j) \\ &= \underline{\underline{0.15 - 0.05j\ \Omega^{-1}}} = \frac{\sqrt{10}}{20}\exp(j\arctan(-1/3))\ \Omega^{-1} \end{aligned}$$

Solution II

Alternatively, we can derive the transfer function using the symbolic representation and substitute the values only at the end.

$$\frac{1}{Z_\parallel} = \frac{1}{Z_C} + \frac{1}{Z_L} + \frac{1}{R_L} = j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}$$

As above, we can write the output voltage using the potential divider equation:

$$v_{out} = \frac{Z_{\|}}{R + Z_{\|}} v_{in}$$

$$H(\omega) = \frac{i_{out}}{v_{in}} = \frac{v_{out}}{R_L} \cdot \frac{1}{v_{in}} = \frac{Z_{\|}}{R_L(R + Z_{\|})} v_{in} \frac{1}{v_{in}} = \frac{1}{R_L\left(\frac{R}{Z_{\|}} + 1\right)} = \frac{1}{R_L\left(j\omega RC + \frac{R}{j\omega L} + \frac{R}{R_L} + 1\right)}$$

$$= \frac{1}{(R_L + R) + jRR_L\left(\omega C + \frac{1}{\omega L}\right)} = \frac{(R_L + R) - jRR_L\left(\omega C + \frac{1}{\omega L}\right)}{(R_L + R)^2 + R^2 R_L^2\left(\omega C + \frac{1}{\omega L}\right)^2}$$

We can sustitute ω_0 to find the same result as in “Solution I”:

$$H(\omega_0) = \frac{6 - j \cdot 5\left(2 \cdot 0.7 - \frac{1}{2 \cdot 0.5}\right)}{6^2 + 5^2\left(2 \cdot 0.7 - \frac{1}{2 \cdot 0.5}\right)^2} = \frac{6 - j \cdot 5\,(1.4 - 1)}{6^2 + 5^2\,(1.4 - 1)^2} = \frac{6 - 2 \cdot j}{6^2 + 2^2} = \frac{6 - 2 \cdot j}{40} = \underline{\underline{0.15 - 0.05 \cdot j\ \Omega^{-1}}}$$

Point c)

This requires applying the given transfer function correctly, and afterwards working out the result in complex numbers algebra.

$$i_{\text{out}} = H(3)v_{\text{in}} \overset{Re}{=} \big(0.07 - 0.08j\big)\big(-\sqrt{2}e^{3jt + \frac{\pi}{2}}\big) \overset{Re}{=} \Big(\frac{\sqrt{113}}{100}e^{-j\arctan\frac{8}{7}}\Big)\big(\sqrt{2}e^{j\pi}e^{3jt + \frac{\pi}{2}}\big)$$

$$\overset{Re}{=} \frac{\sqrt{226}}{100}e^{j[3t + \frac{3\pi}{2} - \arctan\frac{8}{7}]} \overset{Re}{=} \underline{\underline{\frac{\sqrt{226}}{100}\cos\Big(3t + \frac{3\pi}{2} - \arctan\frac{8}{7}\Big)\ \text{A}}} \approx 0.150\cos(3t + 3.86\ \text{rad})\ \text{A}$$

(degree $= 221^o$)
Either the exact answer or the approximate answer (to reasonable number of significant figures) are awarded full marks. Units are not optional.

Point d)

Since the voltage input is a sum of two inputs with different frequencies, then by the superposition principle we can consider the contribution to i_{out} for each term separately. If part b) was done correctly, then we use the two transfer functions.

$$H(3) = 0.07 - 0.08j = \frac{\sqrt{113}}{100}e^{-j\arctan\frac{8}{7}}\ \Omega^{-1}$$

$$H(1) = 0.11 + 0.75j = \frac{13\sqrt{34}}{100}e^{j\arctan\frac{75}{11}}\ \Omega^{-1}$$

and then apply these magnitudes and phase differences to the input voltage terms separately.

$$i_{\text{out}} = \underline{\underline{\frac{\sqrt{110}}{50}\cos\left(3t - \arctan\frac{8}{7}\right) + \frac{13\sqrt{34}}{100}\cos\left(t + \arctan\frac{75}{11}\right)\ \text{A}}}$$

$$\approx 0.213\cos(3t - 0.852) + 0.758\cos(t + 1.43)\ \text{A}$$

Note that if any of the arctan’s are evaluated in degrees, then unless the t-prefactor frequencies have been converted to degrees/sec, then the answer is *wrong*.

Remarks

This question is in complexity and type of tasks comparable to top problem from week 3. Finding limits by inspection and only reasoning as in subquestion a) was extensively practiced during the lectures.

Problem 3 (22 points)

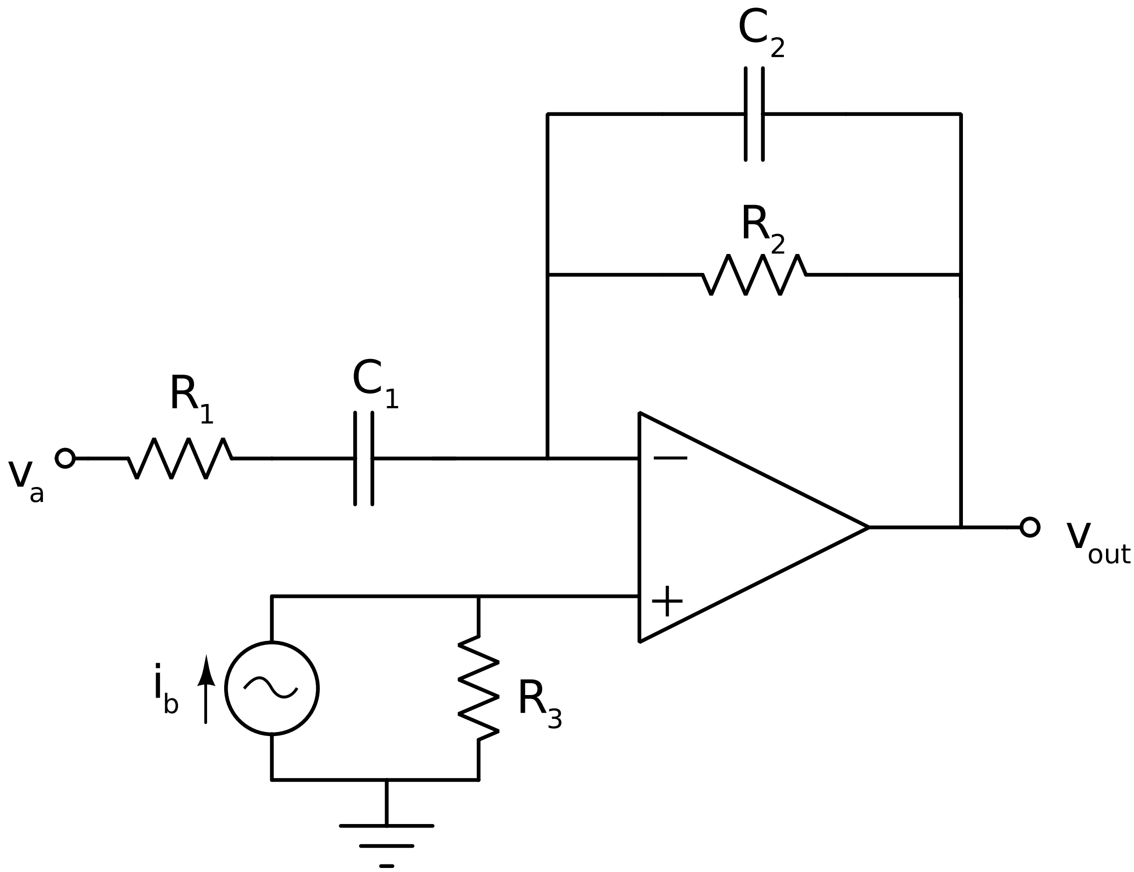


Figure 6: Circuit with OPAMP.

Consider the circuit in Figure 6. The input voltage v_a and input current i_b operate in a **sinusoidal** regime **with the same frequency** ω_0. All components are ideal components. Use parameters $R_1 = 700\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 1\ k\Omega$, $C_1 = C_2 = 10\ \mu F$, $\omega_0 = 1000\ rad\ s^{-1}$.

By the **superposition principle**, v_{out} can be written as

$$v_{\text{out}} = H_1(\omega) \cdot v_a + H_2(\omega) \cdot i_b$$

(a) (8 points) Using the parameters provided above, **calculate** the partial transfer function $H_1(\omega_0)$.

(b) (8 points) Similarly, **calculate** the partial transfer function $H_2(\omega_0)$.

From now on take $|\mathbf{i_b}| = \mathbf{0\ A}$.

(c) (4 points) What is the gain $\left|\frac{v_{\text{out}}}{v_a}\right|$ as $\omega \to 0$? What about $\omega \to \infty$?

(d) (2 points) Assuming $v_a(t) = \cos(\omega_0 t)$ V, determine $v_{\text{out}}(t)$.

Problem 3 - Solution

Part a)

1. Principle of superposition → consider only v_a as a source, consider i_b to be an open circuit.
2. Op amp ideal → no current into/out of input terminals, → voltage at + terminal $v_+ = 0$.
3. Since op amp ideal and in negative feedback, $v_- = v_+ = 0$.

Now what would be quite useful is to find the impedance of the R_1 and C_1 in series, and R_2 and C_2 in parallel. We denote them

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = 700 - \frac{j}{1000 \cdot 10\mu F} = (700 - 100j)\ \Omega$$

and

$$Z_2 = R_2 \parallel Z_{C_2} = \left[\frac{1}{R_2} + j\omega C_2\right]^{-1} = \left[\frac{1}{100} + j \cdot 1000 \cdot 10\mu F\right]^{-1} = \left[\frac{1}{100} + \frac{1}{100}j\right]^{-1} = (50 - 50j)\ \Omega$$

Alternatively, without directly substituting values:

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega C_1 R_1}{j\omega C_1} = \frac{C_1 R_1 \omega - j}{C_1 \omega}$$

and

$$Z_2 = R_2 \parallel Z_{C_2} = \left[\frac{1}{R_2} + j\omega C_2\right]^{-1} = \frac{R_2}{1 + j\omega R_2 C_2}$$

Now we apply the superposition principle to the top half of our circuit and express v_- using the potential divider equation.

$$\begin{aligned}
v_- &= \frac{Z_2}{Z_1 + Z_2} v_a + \frac{Z_1}{Z_1 + Z_2} v_{out} = 0 \\
\frac{Z_1}{Z_1 + Z_2} v_{out} &= -\frac{Z_2}{Z_1 + Z_2} v_a \\
v_{out} &= -\frac{Z_2}{Z_1} v_a \\
\Rightarrow H_1(\omega_0) &= -\frac{Z_2}{Z_1} = -\frac{50 - 50j}{700 - 100j} = \underline{\underline{-0.08 + 0.06j}}
\end{aligned}$$

or substituting numerical values later:

$$\begin{aligned}
H_1(\omega_0) &= -\frac{Z_2}{Z_1} = -\frac{\frac{R_2}{1 + j\omega R_2 C_2}}{\frac{C_1 R_1 \omega - j}{C_1 \omega}} = -\frac{R_2}{1 + j\omega R_2 C_2} \frac{C_1 \omega}{C_1 R_1 \omega - j} = -\frac{C_1 R_2 \omega}{C_1 R_1 \omega + C_2 R_2 \omega + j(R_1 R_2 C_1 C_2 \omega^2 - 1)} \\
&= -\frac{1}{7 + 1 + j(7 - 1)} = \frac{1}{8 + 6j} = -\frac{1}{100}(8 - 6j) = \underline{\underline{-0.08 + 0.06j}}
\end{aligned}$$

Note that the solution may be found without using the voltage divider equation, by just equating the current through the Z_1 and Z_2 components:

$$\frac{v_a - 0}{Z_1} + \frac{v_{out} - 0}{Z_2} = 0 \implies \frac{v_{out}}{Z_2} = -\frac{v_a}{Z_1} \Rightarrow H_1(\omega) = -\frac{Z_2}{Z_1}$$

Part b)

1. Principle of superposition → consider only i_b as a source, $v_a = 0$.
2. Op amp ideal → no current into/out of input terminals, → all current i_b goes through R_3 → voltage at + terminal $v_+ = i_b R_3$.
3. Since op amp ideal and in negative feedback, $v_- = v_+ = i_b R_3$.

Solution I

Consider the new voltage divider circuit "$Ground - [Z_1] - i_b R_3 - [Z_2] - v_{\text{out}}$":

$$i_b R_3 = v_{\text{out}} \frac{Z_1}{Z_1 + Z_2} \implies H_2(\omega_0) = R_3 \frac{Z_1 + Z_2}{Z_1} = 1000 \cdot \big(1.08 - 0.06j\big)\ \text{VA}^{-1} = \underline{\underline{(1080 - 60j)\ \text{VA}^{-1}}}$$

Solution II

$$H_2(\omega_0) = R_3 \frac{Z_1 + Z_2}{Z_1} = R_3(1 + \frac{Z_2}{Z_1}) = R_3(1 - H_1(\omega_0)) = R_3 \left(1 + \frac{C_1 R_2 \omega}{C_1 R_1 \omega + C_2 R_2 \omega + j(R_1 R_2 C_1 C_2 \omega^2 - 1)}\right)$$
$$= 1k(1 + 0.08 - 0.06j)\ \text{VA}^{-1} = \underline{\underline{(1080 - 60j)\ \text{VA}^{-1}}}$$

Solution III

By using KCL at v_-:

$$0 = \frac{0 - v_-}{Z_1} + \frac{v_{out} - v_-}{Z_2} \iff \frac{v_{out}}{Z_2} = v_- \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) \iff v_{out} = \frac{Z_1 + Z_2}{Z_1} v_-$$

Part c)

For $\omega \to 0$, $Z_{C_1} \to \infty$, so $Z_1 \to \infty$ while $Z_2 \to R_2$. Therefore from part a), we see that

$$H_1 \propto -\frac{Z_2}{Z_1} \to -\frac{R_2}{\infty} = 0$$

and so $v_{\text{out}}/v_a \to 0$.

For $\omega \to \infty$, $Z_{C_1} \to 0$, so $Z_1 \to R_1$ while $Z_2 \to 0$. Therefore from part a) we see that

$$H_1 \propto -\frac{Z_2}{Z_1} \to \frac{0}{R_1} = 0$$

and so $v_{\text{out}}/v_a \to 0$.

Part d)

We know that $H_1(\omega_0) = -0.08 + 0.06j = \frac{1}{10} e^{j\left[\frac{\pi}{2} + \arctan\frac{4}{3}\right]}$, taking care of the fact that we are in the positive Im, negative Re quadrant.
Then the answer is given by:

$$v_{\text{out}}(t) = \underline{\underline{\frac{1}{10}\cos\left(\omega_0 t + \frac{\pi}{2} + \arctan\frac{4}{3}\right)}}\ V \approx \frac{1}{10}\cos(\omega_0 t + 2.50\text{rad})\ V$$

Remarks

Question a) is comparable to the integrator circuit which was part of the top problems in week 4. Part b) is similar but simpler because v_{in} does not contribute. Similarly as for problem 2, solving limits by inspection was practiced in the lectures.

Problem 4 (19 Points)

Legend:

$$A \cdot B = \text{AND}$$

$$A + B = \text{OR}$$

$$\overline{A} = \text{NOT A}$$

(a) **(8 points) Logic minimisation with Karnaugh maps**

$$Y = (\overline{D} \cdot \overline{C} \cdot \overline{B} \cdot \overline{A}) + (\overline{D} \cdot C \cdot \overline{B} \cdot \overline{A}) + (D \cdot C \cdot \overline{B} \cdot \overline{A}) + (D \cdot C \cdot \overline{B} \cdot A) + (D \cdot C \cdot B \cdot A)$$
$$+ (D \cdot C \cdot B \cdot \overline{A}) + (D \cdot \overline{C} \cdot B \cdot A) + (\overline{D} \cdot \overline{C} \cdot B \cdot \overline{A}) + (\overline{D} \cdot C \cdot B \cdot \overline{A})$$

Using a Karnaugh map, derive an optimised expression to implement the above logical function.

(b) **(3 points) Logic mapping and minimisation with Boolean algebra**

Convert the circuit shown in Fig. 7 into **written Boolean form**. Use Boolean algebra to **simplify** the equation and show that the final result is equal to the following expression:

$$Y = (A \cdot B \cdot (C + D)) + (\overline{A} \cdot (\overline{B} + \overline{D}))$$

(c) **(8 points) NAND logic**

Convert the simplified expression provided in part b) to NAND logic using Boolean algebra. Use as many **2-input NAND gates** (NAND2) as needed (no other kind of gates are allowed) to draw the circuit to implement the combinational logic.

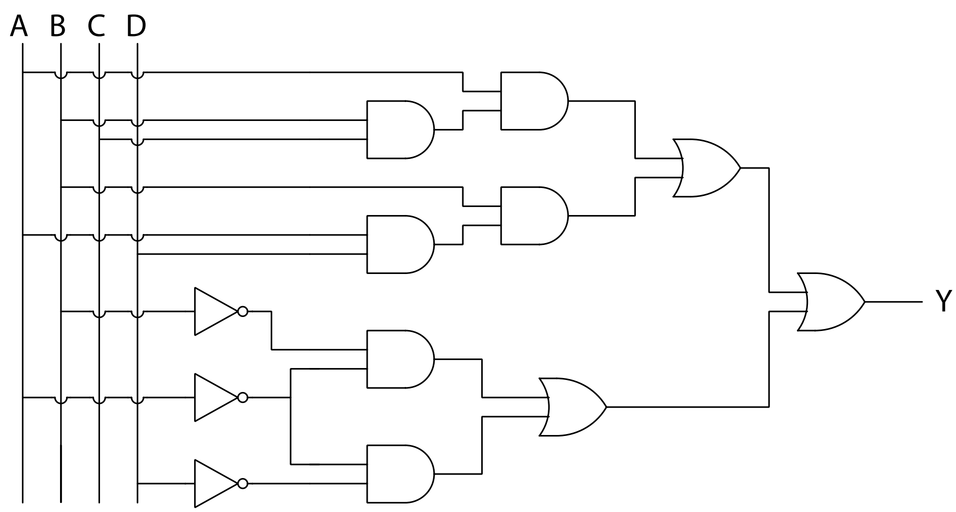
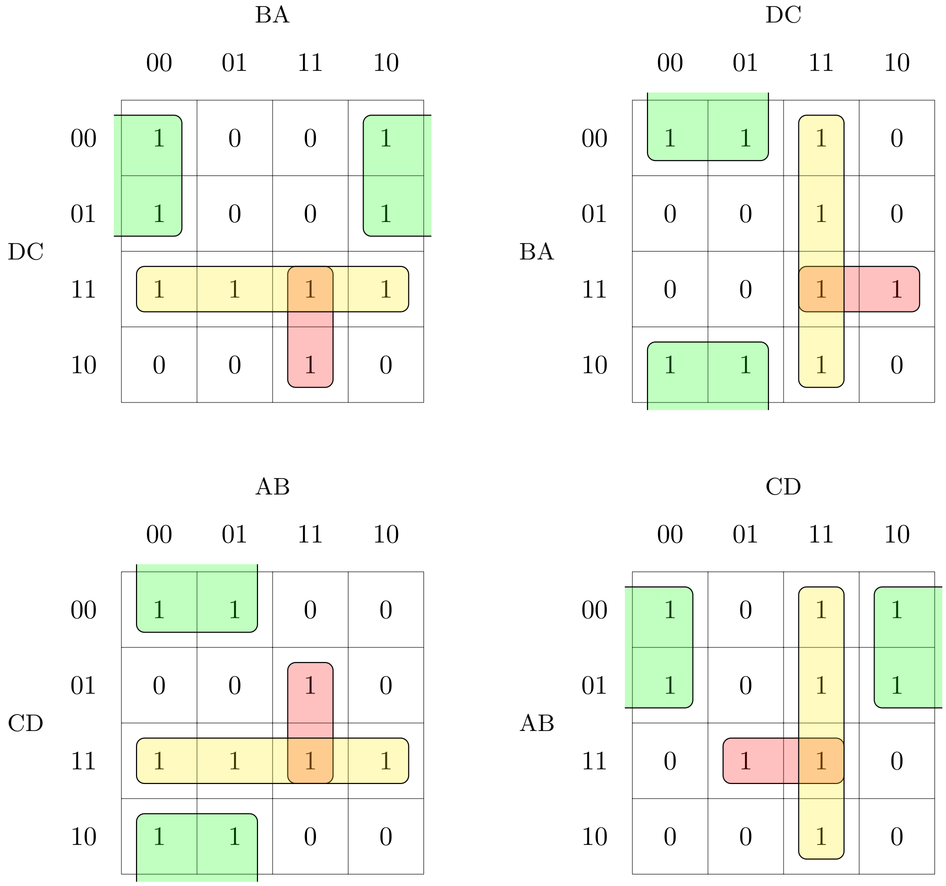


Figure 7: Logic circuit.

Problem 4 - Solution

Point a)

The Karnaugh map is drawn with gray code, so only one digit changes per step, with 2 variables vertical and 2 horizontal. Each of the given terms (e. g. $\overline{D} \cdot \overline{C} \cdot A \cdot \overline{B}$) appears as a one in the map (e.g. in the cell with $B = C = D = 0$ and $A = 1$). All other positions are filled with a zero. Mark all groups of 16 (all variables reduced), 8 (3 variables reduced), 4 (two variables reduced), 2 (one variable reduced) that you can find, individual positions can be used in multiple groups. Remove any redundant group. Groups can be formed over the edge.



$$y = (green) + (yellow) + (red)$$

$$y = (\overline{A} \cdot \overline{D}) + (C \cdot D) + (A \cdot B \cdot D)$$

$$A \cdot B = \text{AND}$$

$$A + B = \text{OR}$$

$$\overline{A} = \text{NOT A}$$

Remarks

The Karnaugh map for this question is comparable to the one provided in the solution of the top problem of Week 8, as it is the task to find the reduced formula.

Point b)

First write the formula:

$$y = (A \cdot B \cdot C) + (B \cdot A \cdot D) + (\overline{B} \cdot \overline{A}) + (\overline{A} \cdot \overline{D})$$

Now use the rule of distributive law to first extract $A \cdot B$ and then $\overline{A}$

$$y = (A \cdot B \cdot (C + D)) + (\overline{B} \cdot \overline{A}) + (\overline{A} \cdot \overline{D})$$

$$y = (A \cdot B \cdot (C + D)) + (\overline{A} \cdot (\overline{B} + \overline{D}))$$

Point c)

$$y = (A \cdot B \cdot (C + D)) + (\overline{A} \cdot (\overline{B} + \overline{D}))$$

The De Morgan's laws can be used to turn all gates into 2-input NAND gates, after the first step of logic simplification. First change the formula to only have 2-input gates

$$y = ((A \cdot B) \cdot (C + D)) + (\overline{A} \cdot (\overline{B} + \overline{D}))$$

Introduce double inversion to every inner 2-input OR expressions:

$$y = ((A \cdot B) \cdot \overline{\overline{(C + D)}}) + (\overline{A} \cdot \overline{\overline{(\overline{B} + \overline{D})}})$$

Swap gates from 2-input OR gates to 2-input NAND gates by inverting the inputs:

$$y = ((A \cdot B) \cdot \overline{(\overline{C} \cdot \overline{D})}) + (\overline{A} \cdot \overline{(\overline{\overline{B}} \cdot \overline{\overline{D}})})$$

Remove obsolete double inversion on single variables:

$$y = ((A \cdot B) \cdot \overline{(\overline{C} \cdot \overline{D})}) + (\overline{A} \cdot \overline{(B \cdot D)})$$

Add double inversion to the outer 2-input OR gate and swap to NAND2:

$$y = \overline{\overline{((A \cdot B) \cdot \overline{(\overline{C} \cdot \overline{D})}) + (\overline{A} \cdot \overline{(B \cdot D)})}}$$

$$y = \overline{\overline{((A \cdot B) \cdot \overline{(\overline{C} \cdot \overline{D})})} \cdot \overline{(\overline{A} \cdot \overline{(B \cdot D)})}}$$

Convert the last remaining AND2 to NAND2 by adding double inversion:

$$y = \overline{\overline{(\overline{\overline{(A \cdot B)}} \cdot \overline{(\overline{C} \cdot \overline{D})})} \cdot \overline{(\overline{A} \cdot \overline{(B \cdot D)})}}$$

Draw the circuit, use a NAND2 with both inputs connected together to implement a NOT gate. The resulting circuit is shown in Fig. 8.

Remarks

The use of the Morgan's laws for the translation of arbitrary logic functions to NAND and NOR logic was extensively covered in the lecture (both in graphical and algebraic form). Top problem from Week 7 has covered a similar problem, providing more practice with Boolean algebra operations.

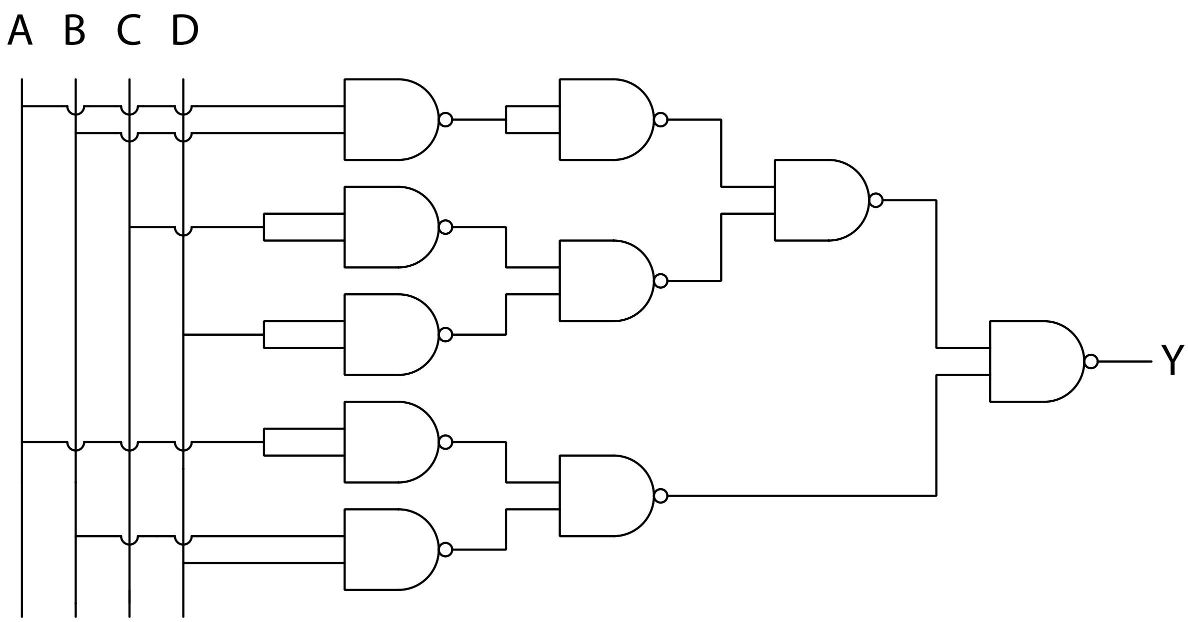


Figure 8: The 2-input NAND circuit.

Electronics and Signal Processing - Formula Sheet

Ohm's law: $V = ZI$ Capacitors: $V = \frac{Q}{C} = \frac{1}{C}\int I \, \mathrm{d}t$ Inductors: $V = L\,\frac{\mathrm{d}I}{\mathrm{d}t}$

Complex impedance: $Z_R = R$, $Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$ Reactance: $X = \mathfrak{Im}(Z)$

Quality ratio : $Q = \frac{f_0}{B}$ (for bandwidth B and resonance frequency f_0)

Root-mean-square voltage of sinusoidal signal = 0.707 of amplitude

Voltage gain: $A_v = \frac{V_o}{V_i}$ dB voltage gain = $20\log_{10}(\frac{V_o}{V_i})$

Closed loop gain: $G = \frac{A}{1+AB}$, where A is the forward gain and B is the feedback gain.

Output voltage of an OpAmp: $V_{out} = A\,(V_+ - V_-)$

Characteristic impedance of a cable: $Z_{eq} = \sqrt{\frac{r}{2\omega c}}(1-j)$ (RC cable), $Z_{eq} = \sqrt{\frac{l}{c}}$ (LC cable)

Speed of signal in a cable: $v = \frac{1}{\sqrt{lc}}$ (LC cable)

Effective impedance seen by a source connected to a cable (length Λ, Z_0) and a load with impedance Z: $Z_{eff} = Z_0\frac{Z - jZ_0 tan(k\Lambda)}{Z_0 - jZ tan(k\Lambda)}$, where $k = \frac{2\pi}{\lambda} = \omega\sqrt{lc}$

Boolean Algebra

- Commutative laws: $AB = BA$, $A + B = B + A$
- Distributive laws: $A(B + C) = AB + AC$, $A + BC = (A + B)(A + C)$
- Associative laws: $A(BC) = (AB)C$, $A + (B + C) = (A + B) + C$
- Absorption law: $A + AB = A\,(A + B) = A$
- De Morgan's laws: $\overline{A + B} = \overline{A}\cdot\overline{B}$, $\overline{AB} = \overline{A} + \overline{B}$
- Other: $A + \overline{A}\cdot B = A + B$, $A\,(\overline{A} + B) = AB$

Complex Numbers Algebra

$|z| = \sqrt{\mathfrak{Re}(z)^2 + \mathfrak{Im}(z)^2}$

$e^{j\phi} = \cos(\phi) + j\sin(\phi) \implies e^{j\frac{\pi}{2}} = j$, $e^{-j\frac{\pi}{2}} = -j$, $e^{j\pi} = -1 = j^2$

$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$ $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$